

# DISTRIBUTED PRIMARY USER IDENTIFICATION FROM IMPRECISE LOCATION INFORMATION

*Pavle Belanovic, Sergio Valcarcel Macua, Santiago Zazo*

Universidad Politécnica de Madrid  
Escuela Técnica Superior de Ingenieros de Telecomunicación  
Email: pavle@gaps.ssr.upm.es

## ABSTRACT

We study a cognitive radio scenario in which the network of secondary users wishes to identify which primary user, if any, is transmitting. To achieve this, the nodes will rely on some form of location information. In our previous work we proposed two fully distributed algorithms for this task, with and without a pre-detection step, using propagation parameters as the only source of location information. In a real distributed deployment, each node must estimate its own position and/or propagation parameters. Hence, in this work we study the effect of uncertainty, or error in these estimates on the proposed distributed identification algorithms. We show that the pre-detection step significantly increases robustness against uncertainty in nodes' locations.

**Index Terms**— cognitive radio, distributed systems, wireless sensor networks, detection, consensus.

## 1. INTRODUCTION

Radio spectrum is the scarcest resource in modern wireless communication systems, and it is no surprise that a growing number of cognitive radio schemes has appeared in recent years to try to increase the efficiency of its use.

A fundamental problem in any cognitive radio scheme is the detection of unused resources in the time-frequency plane. This is equivalent to detecting and identifying active primary users, who can occupy the spectrum at will. We study the identification of which, if any, primary user is transmitting, by a network of secondary nodes without a fusion center and with only elementary location information.

In a network of decision makers, distributed detection has been thoroughly studied and different solutions have been proposed. The problem is to decide what information the agents should share, and to find optimal fusion rules to combine the local outputs. Decentralized binary detection [1, 2, 3] (and the references therein) proposes a parallel architecture in which every node sends a summary of its own observations (e.g. quantized values, test outputs or hard decisions) to a fusion center in charge of making the final decision. Recently, completely distributed implementations, in which there is no fusion

center (so the nodes have to collaborate with each other to converge to the global solution) have also appeared; paying special attention to on-line algorithms in which nodes collaborate and detect in the same timescale [4, 5, 6].

The M-ary hypothesis testing, in particular with no prior knowledge of the probability distributions of the alternative hypothesis, has received much less attention. A number of decentralized approaches, which rely on a fusion center, have been proposed. For instance, [7] applies a blind algorithm after estimating the prior probabilities of the hypothesis; while in [8] the M-ary detection problem is converted into a sequence of binary detection problems. A fully distributed scheme based on belief propagation has been proposed in [9], but it requires knowledge of the prior probabilities in order to maximize the posterior distribution.

In [10], we presented two fully distributed schemes for the detection and identification of primary user activity, with each node given only the knowledge of the noise statistic and the attenuation factors to that node from each primary user. However, this knowledge is assumed to be perfect, and in this contribution we relax the later assumption to include error in the estimates of the attenuation factors.

## 2. PROBLEM DEFINITION

Let  $P$  primary and  $S$  secondary nodes be uniformly randomly distributed in a given geometric area. Each primary user may transmit at any time, and we assume that at most one primary user is transmitting at any time. The transmission pattern is "bursty", so that if the primary user  $p$  is transmitting, the signal  $s_p$  alternates between an *active* and a *passive* phase. The length of each state is a Poisson random variable with parameters  $\lambda q$  and  $(1 - \lambda)q$  respectively, so that  $\lambda$  is the activity factor and  $q$  is the expected number of samples in each full cycle. In the active state,  $s_p \sim \mathcal{N}(0, \sigma_t^2)$ , and in the passive state  $s_p = 0$ . For each transmission, the primary node selects random  $\sigma_t^2$  and  $\lambda$ , unknown to any secondary node.

Furthermore, we assume a simple and general propagation model,  $x_s = \alpha_{ps}s_p + n$ , where  $x_s$  is the signal received at the secondary node  $s$ ,  $\alpha_{ps}$  is the attenuation of the signal, and  $n \sim \mathcal{N}(0, \sigma_n^2)$  is additive white Gaussian noise (AWGN), with iid realizations at all the secondary nodes. We assume that each node is capable of estimating  $\sigma_n$  perfectly.

It is important to note that this propagation model is generic. Although it does assume temporally static attenuations, it does not assume any properties on the attenuations spatially - in terms of correlations among the nodes, or lack thereof. In fact, the approach presented in this paper is completely independent of how the matrix of attenuations  $\mathbf{A} = [\alpha_{ps}]_{P \times S}$  is formed.

---

This work was supported in part by the Spanish Ministry of Science and Innovation under the grant TEC2009-14219-C03-01; the Spanish Ministry of Science and Innovation in the program CONSOLIDER-INGENIO 2010 under the grant CSD2008-00010 COMONSENS; the European Commission under the grant FP7-ICT-2009-4-248894-WHERE-2; the European Commission under the grant FP7-ICT-223994-N4C and the Spanish Ministry of Science and Innovation under the complementary action grant TEC 2008-04644-E; Spanish Ministry of Science and Innovation under the grant TEC2010-21217-C02-02-CR4HFDVL.

What is important however is the knowledge of  $\mathbf{A}$  by the secondary nodes. We assume that each node  $s$  has some degree of knowledge of its corresponding column of  $\mathbf{A}$ . In [10] the assumption was that this knowledge is perfect. As will be discussed below, in this contribution we relax this assumption to consider the effects of imperfect knowledge of these attenuation factors.

### 2.1. Identification schemes

Once the nodes receive the transmitted signal, they carry out one of the two distributed identification schemes presented in [10] in order to identify the primary user which is transmitting, if any. This is in fact a hypothesis testing problem, with  $P + 1$  possibilities  $\{\mathcal{H}_0, \mathcal{H}_1, \dots, \mathcal{H}_P\}$ , where  $\mathcal{H}_0$  represents no transmission from any primary. Both schemes are fully distributed and do not rely on a fusion center, thus lending them robustness and scalability.

Our first scheme performs direct identification based on distributed hypothesis testing. Each node  $s$  in isolation performs energy sampling, where  $W$  integration windows, each of length  $L$  samples, produces an energy estimate  $y_s[w] = \frac{1}{L} \sum_{l=1}^L (x_s[l])^2$ , with  $w \in \{1, 2, \dots, W\}$ . Using the knowledge of the noise statistic, we generate a new variable  $z_s = y_s - \sigma_n^2$ , distributed as

$$z_s \sim \begin{cases} \mathcal{N}\left(0, \frac{2\sigma_n^4}{L}\right) & \mathcal{H}_0 \\ \mathcal{N}\left(\sigma_{ps}^2 \lambda, \frac{2}{L}(\sigma_n^4 + \sigma_{ps}^4 \lambda + 2\sigma_n^2 \sigma_{ps}^2 \lambda)\right) & \mathcal{H}_p \end{cases}$$

where  $\sigma_{ps} = \alpha_{ps} \sigma_t$  and  $p \in \{1, 2, \dots, P\}$ . Each node  $s$  then constructs  $P + 1$  hypotheses to test, by compensating its own received distribution of  $W$  samples of  $z_s$  exactly  $P + 1$  times. The first compensation represents  $\mathcal{H}_0$ , i.e. the possibility that  $z_s$  contains only noise energy, and is constructed simply by using the raw data itself (no compensation). The following  $P$  compensations are performed by multiplying the received distribution by a compensation factor  $\beta_p \alpha_{ps}^{-2}$ , i.e. one compensated distribution for each possible primary node. The factor  $\beta_p$  serves to normalize each of the hypothesis, relative to  $\mathcal{H}_0$ , so that later on their variances will be directly comparable. Hence,  $\beta_p = (\|\alpha_p\|)^{-1/2}$ , where  $\alpha_p = [\alpha_{p1}, \alpha_{p2}, \dots, \alpha_{pS}]^T$ . Hence, there are  $S$  distributions for each hypothesis, one per node. We note that for the correct hypothesis all the nodes agree perfectly in the mean, but not in the variance.

Estimating which hypothesis is true is the next challenge, and the first to use coordination among the secondary nodes. An intuitive approach would be to choose the hypothesis with a minimum sum of distances among the  $S$  distributions. It is easy to show that this sum of distances is proportional to the sample variance of the set of  $S$  compensated means. Hence, the problem reduces to finding the hypothesis with minimum variance across the sample means from the  $S$  nodes. This problem is easily tackled in a distributed fashion using averaging consensus algorithms following the idea of constructing the sample covariance matrix shown in [11].

In a low SNR regime the mode of failure of the identification-only scheme above, and indeed any identification scheme, is that of always choosing  $\mathcal{H}_0$ , even when a signal is present, simply because this signal is too weak to identify a particular primary node transmitting. Hence, it makes sense to perform an (optimal or nearly-optimal) detection step first, detecting the activity of *any* primary node, followed then by an identification procedure similar to that above, but this time with only  $P$ , rather than  $P + 1$  hypotheses.

If each node  $s$  had the knowledge of all the parameters of its  $z_s$ , the optimal test based on the Neyman-Pearson criterion would be given by  $T(z_s) = \eta z_s^T \mathbf{1} + \theta z_s^T z_s$ . Both  $\eta$  and  $\theta$  are functions of the statistics of  $z_s$ , and since these are not known, the approach

is not feasible. Therefore, we proposed a sub-optimal approach  $T(z_s) = z_s^T z_s$  so that

$$T(z_s) \sim \begin{cases} \mathcal{N}\left(\frac{2W\sigma_n^4}{L}, \frac{8W\sigma_n^8}{L^2}\right) & \mathcal{H}_0 \\ \mathcal{N}\left(W(\sigma_a^2 + \sigma_{ps}^4 \lambda^2), 2W\sigma_a^2(\sigma_a^2 + 2\sigma_{ps}^4 \lambda^2)\right) & \mathcal{H}_p \end{cases}$$

where  $\sigma_a^2 = \frac{2}{L}(\sigma_n^4 + \sigma_{ps}^4 \lambda + 2\sigma_n^2 \sigma_{ps}^2 \lambda)$  for compactness. One obvious advantage is that this test does not depend on the estimates of  $\sigma_{ps}$  and  $\lambda$ .

The threshold is hence  $\gamma = \frac{2\sigma_n^4}{L}(\sqrt{2W}Q^{-1}(P_{fa}) + W)$ , and is calculated by each node in isolation, for a defined probability of false alarm  $P_{fa}$ , where  $Q^{-1}(\cdot)$  is the inverse Q-function.

Typically each node would compare the local  $T(z_s) \leq \gamma$  producing a 1-bit detection decision, which are then combined globally. Instead, we propose a *weighted* global test  $T_* = \frac{1}{S} \sum_{i=1}^S (T(z_s) - \gamma) \leq 0$ . Although the factor  $S^{-1}$  is quite unnecessary, it shows that this global value can also be derived in a distributed fashion via average consensus. If each node calculates its *vote* as a degree of confidence  $T(z_s) - \gamma$ , simply the sign (+ or -) of the global average of the votes (available at all the nodes simultaneously) is the outcome of the global test. This weighing allows the nodes closer to any transmitting primary to exert a bigger influence, as desired.

Once the detection stage is performed in this distributed fashion, all the nodes can carry out the identification procedure (also distributed) as shown earlier, but this time with  $P$  rather than  $P + 1$  hypotheses.

### 3. IMPRECISE LOCATION INFORMATION

So far the location information available in each secondary node  $s$ , being the attenuation factors  $\alpha_{ps}$  from each primary node  $p$ , were assumed to be known perfectly. Consequently, the compensation of the received samples by the different  $\alpha_{ps}$  is perfect as well. In turn, this means that the correct hypothesis in theory aligns perfectly in all the nodes, perturbed only by the AWGN noise in the system [10].

In this paper we consider the effect of imperfect knowledge of the attenuation factors. Hence, each  $\alpha_{ps}$  will be modeled as a random variable. To derive its distribution, we first look at how each node can obtain/estimate its set of attenuation factors. The most likely sources, in a realistic system, would be on-line calibration, or direct estimation from a geometric model. We will focus on the latter approach to derive a suitable probability density function (PDF) for  $\alpha_{ps}$ .

Let us assume a 2-dimensional scenario in which a secondary node  $s$ , located at  $\mathbf{x}_s$ , is trying to estimate its *voltage* attenuation factor  $\alpha$  for a primary node  $p$ , located at  $\mathbf{x}_p$ . The basis for this process is to determine the distance between the nodes,  $d = \|\mathbf{x}_p - \mathbf{x}_s\|$ , and from this apply a propagation model to derive  $\alpha$ . For simplicity let us consider the free space propagation model, in which  $\alpha \propto d^{-1}$ , or

$$\alpha = cd^{-1} \quad (1)$$

where  $c$  is a constant that encapsulates all the effects other than the distance  $d$ .

Now, as proposed, let us assume that  $s$  does not have perfect knowledge of its location  $\mathbf{x}_s$ , but rather an estimate  $\tilde{\mathbf{x}}_s = \mathbf{x}_s + \epsilon$ , as shown in Figure 1.

Assuming isotropic zero-mean iid errors, we conclude that  $\epsilon \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I})$  and  $\tilde{\mathbf{x}}_s \sim \mathcal{N}(\mathbf{x}_s, \sigma^2 \mathbf{I})$ . From here  $s$  estimates its distance to  $p$  as  $\tilde{d} = \|\mathbf{x}_p - \tilde{\mathbf{x}}_s\|$ , and it is straight-forward to show that  $\tilde{d} \sim \text{Rice}(d, \sigma)$ .

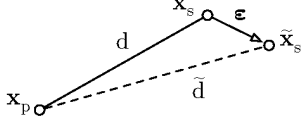


Fig. 1. Error in a node's position

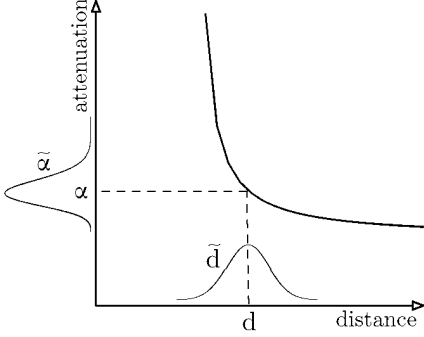


Fig. 2. Error transformation by the free space model

Given the random estimate  $\tilde{d}$ ,  $s$  produces an estimate of the random attenuation factor  $\tilde{\alpha}$  as shown in Figure 2. From (1), we see that the PDF of  $\tilde{\alpha}$  is given by

$$f(\tilde{\alpha}|d, \sigma) = \frac{c^3}{|c|\tilde{\alpha}^3\sigma^2} \exp\left(-\frac{\left(\frac{c^2}{\tilde{\alpha}^2} + d^2\right)}{2\sigma^2}\right) I_0\left(\frac{dc}{\tilde{\alpha}\sigma^2}\right) \quad (2)$$

where  $I_0$  is the zero-order modified Bessel function of the first kind.

This result allows us to draw random samples of  $\tilde{\alpha}$ , representative of each node being uncertain about its position and estimating the attenuation factors from this imprecise information.

#### 4. EXPERIMENTS

Given (2), we are able to carry out simulation experiments with imprecise location information of the two distributed identification schemes described in Section 2.1. In these experiments we explore a scenario with  $P = 4$  primary and  $S = 20$  secondary nodes scattered uniformly over a square area of side 200 m. Both schemes are tested over 20000 independent experiments (i.e. network realizations).

The activity factor is 50%, or  $\lambda = 0.5$ , over an expected value of  $q = 20$  samples per complete cycle. At the receivers, AWGN has  $\sigma_n = 10^{-6}$  and the squared signal is integrated over  $W = 100$  integration windows of  $L = 200$  time samples each.

The parameters which are varied are the  $\sigma_t$ , which influences the signal-to-noise ratio (SNR) at the nodes; and  $\sigma^2$ , which expresses the degree of imprecision of the location information. The range covered is  $\sigma_t \in [10^{-6}, 10^{-1}]$ . Also, we let  $\sigma^2$  vary in proportion to the distance between the primary and secondary node, as is realistic. In other words, the parameter which is actually varied is  $\sigma^2/d$ . The values it assumes are 0 (perfect location information), 0.01, 0.1, 0.3, 0.4, and 1 (variance of the location uncertainty equal to the distance to the primary). Please note that theoretically even higher values of  $\sigma^2/d$  are possible, but would be very extreme and thus unrealistic.

Several conclusions can be drawn from the results shown in Figure 3. First we note by comparing 3(a) and 3(c) that the Neyman-Pearsons pre-detection step introduces a gain of more than a decade in  $\sigma_t$  even in the case of no uncertainty in the nodes' positions, as reported in [10]. Furthermore, since it does not rely on attenuations

$\alpha$ , the pre-detection step is unaffected by uncertainty in nodes' locations. On the other hand, when the pre-detection step is missing, the uncertainty has a very strong effect on the detection performance, as it relies heavily on the knowledge of the attenuation factors  $\alpha$ . We see that in the high-SNR regime in 3(a), when  $\sigma_t$  is sufficiently high that all the nodes hear it clearly, detection saturates to certainty (i.e. 1) only when there is no uncertainty in the node's positions. Otherwise it deteriorates very quickly, so that even a small error, when the variance of uncertainty is 1% of the distance to the primary node, has a strong effect, causing a drop of 30% in detection performance.

This behavior in detection has strong repercussions on the next stage, being the identification of the transmitting primary node.

First, let us note that the identification scheme based solely on attenuation factors (i.e. *without* the pre-detection step) has a particular mode of failure. Say a primary  $i$  is active, and the algorithm commits an identification error. Typically it will be by deciding that *no* primary user is active, rather than mistakenly picking a *different* primary user  $j$ . This is in fact the reason that for this algorithm the curves for detection and identification performance are similar (compare 3(a) and 3(b)). In other words, a typical error for this algorithm is both in detection and identification at the same time.

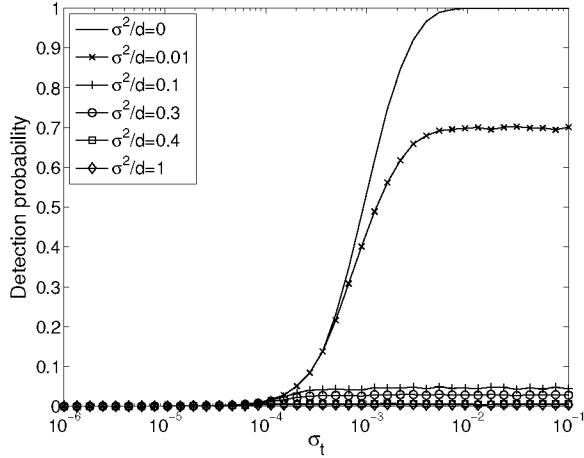
Hence, we can expect that by performing a pre-detection step, and thus removing the option of mistakenly picking no transmitting primary when in fact there is one, we would improve the performance considerably. Indeed, as can be seen by comparing 3(b) and 3(d), in the case of no uncertainty, i.e. perfect location knowledge, the identification performance with the pre-detection step is superior by the same margin as in the detection performance described earlier (more than a decade of improvement in  $\sigma_t$ ).

The same mechanism is still in play when we introduce uncertainty in nodes' locations. It is clear from 3(b) and 3(d) that the identification performance is improved significantly by the introduction of the pre-detection step. In fact, it takes about 40 times grater variance of uncertainty to cause the same degradation in performance. This is again due to the fact that without the pre-detection step, node uncertainty causes the same typical error: choice of no transmitting primary rather than a mistaken identity. This mode of failure is valid even in the high-SNR regime (high values of  $\sigma_t$ ).

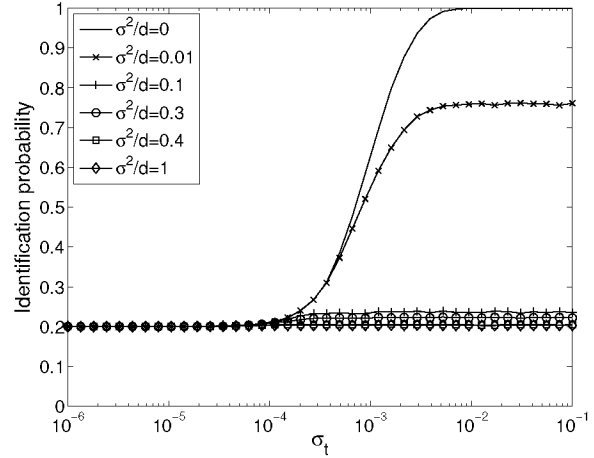
By taking away the option of selecting no transmitting primary (when there is a transmission), the pre-detection step only leaves the option of making a mistake via a mistaken identity. This is of course a much more difficult error to make, due to the shared geometry of the network, and hence requires a much higher level of uncertainty in nodes' locations in order to be able to occur. This effect is clearly seen in 3(d).

#### 5. CONCLUSIONS

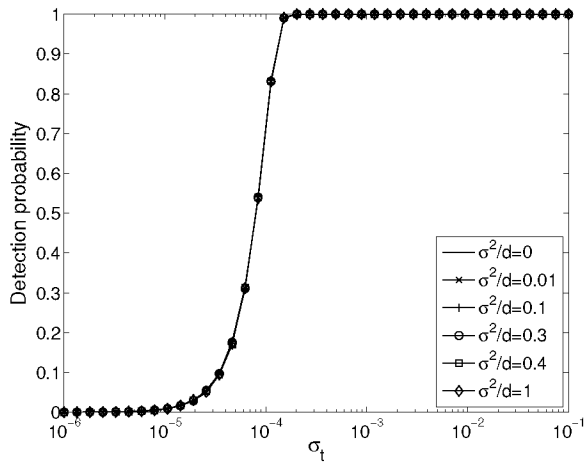
In this contribution we studied the effect of imprecise location information on two fully distributed schemes for the detection of primary user activity in a cognitive radio scenario. We show that if the secondary nodes derive the necessary attenuation factors from a geometric model, isometric iid Gaussian error in the node's own location translates to a reciprocal-of-Rice PDF for the attenuation factor  $\alpha$ , as shown in (2). Experiments show that this uncertainty has a strong effect on the performance of our scheme without pre-detection, rendering it practically unusable. However, the scheme with the pre-detection step is very robust to uncertainty in nodes' locations.



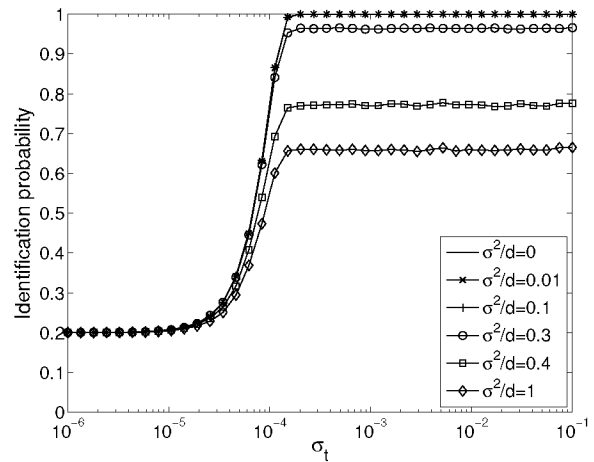
(a) Probability of detection, without pre-detection



(b) Probability of identification, without pre-detection



(c) Probability of detection, with pre-detection



(d) Probability of identification, with pre-detection

**Fig. 3.** Probability of detection and identification as functions of  $\sigma_t$  and  $\sigma^2/d$

## 6. REFERENCES

- [1] Pramod K. Varshney, *Distributed Detection and Data Fusion*, Springer-Verlag, 1997.
- [2] R.S. Blum, S.A. Kassam, and H.V. Poor, "Distributed Detection with Multiple Sensors II. Advanced topics," *Proceedings of the IEEE*, vol. 85, no. 1, pp. 64–79, Jan 1997.
- [3] J.-F. Chamberland and V.V. Veeravalli, "Decentralized Detection in Sensor Networks," *IEEE Trans. on Signal Processing*, vol. 51, no. 2, pp. 407–416, Feb 2003.
- [4] P. Braca, S. Marano, V. Matta, and P. Willett, "Asymptotic Optimality of Running Consensus in Testing Binary Hypotheses," *IEEE Trans. on Signal Processing*, vol. 58, no. 2, pp. 814–825, Feb 2010.
- [5] F. Cattivelli and A. Sayed, "Distributed Detection Over Adaptive Networks Using Diffusion Adaptation," *IEEE Trans. on Signal Processing*, vol. 59, no. 5, pp. 1917–1932, May 2011.
- [6] D. Bajovic, D. Jakovetic, J. Xavier, B. Sinopoli, and J.M.F. Moura, "Distributed Detection via Gaussian Running Consensus: Large Deviations Asymptotic Analysis," *IEEE Trans. on Signal Processing*, vol. 59, no. 9, pp. 4381–4396, Sep 2011.
- [7] Bin Liu, A. Jeremic, and K.M. Wong, "Optimal Distributed Detection of Multiple Hypotheses using Blind Algorithm," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 47, no. 1, pp. 317–331, Jan 2011.
- [8] Qian Zhang and Pramod K. Varshney, "Decentralized M-ary Detection via Hierarchical Binary Decision Fusion," *Information Fusion*, vol. 2, no. 1, pp. 3–16, 2001.
- [9] V. Saligrama, M. Alanyali, and O. Savas, "Distributed Detection in Sensor Networks With Packet Losses and Finite Capacity Links," *IEEE Trans. on Signal Processing*, vol. 54, no. 11, pp. 4118–4132, Nov 2006.
- [10] Pavle Belanovic, Sergio Valcarcel Macua, and Santiago Zazo, "Location-aided Distributed Primary User Identification in a Cognitive Radio Scenario," in *International Conference on Acoustics, Speech and Signal Processing ICASSP*, 2012.
- [11] Sergio Valcarcel Macua, Pavle Belanovic, and Santiago Zazo, "Distributed Linear Discriminant Analysis," in *International Conference on Acoustics, Speech and Signal Processing ICASSP*, 2011.